Demand-Responsive Pricing in Open Wireless Access Markets

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Abstract— Radio resource management (RRM) across operator boundaries is emerging as a salient feature for wireless systems beyond 3G. Until recently, research has been confined to solutions where cooperating networks enter explicit sharing agreements that define how responsibilities and revenues should be divided. An alternative would be to share the infrastructure implicitly by establishing an open wireless access market wherein networks not only compete for users on a long-term time-scale, but also on a much shorter time-base. This could be realized with an architecture where autonomous trade-agents, that reside in terminals and access points (APs), manage the resources through negotiations.

In this paper we develop a framework for studying demand-responsive pricing in contexts where APs with overlapping coverage compete for users. Resources are partitioned through a proportional fair divisible auction and our aim is to establish if, and when, an open market for wireless access can be self-sustained. Compared to a scenario where APs cooperate, our results show that an open access market results in better services at lower price which in the prolonging also yields more satisfied customers. As an effect demand will increase and, from the perspective of the APs, act as a counterbalance to the reduced prices. Thus, the revenue earned by the APs will be comparable to the one in which obtained through AP cooperation and monopoly (cartel) pricing. Generally speaking, the difference between the cooperative and noncooperative RRM is small when the demand is concave and increases with the convexity of demand.

I. INTRODUCTION

Future wireless networks will be composed of a mixture of radio access technologies (RATs). Albeit some of them, such as WLAN and GSM, evidently are complementary, others with more similar capabilities, will act as substitutes. As furthermore no single provider is likely to deploy all standards, infrastructure sharing is one possible alternative to offer high-speed data access in an cost efficient manner. The sharing of wireless infrastructure, however, raises the question of how resources and revenues should be divided when multiple subsystems, managed by potentially competing actors, are involved in delivering the access service. An alternative to “traditional” infrastructure sharing; see e.g. [1] and references therein; would be to establish an “open” access market through which users can access all deployed “access points” (APs), just as in a common market place. Therein users, or rather “selfish” autonomous trade-agents that reside in terminals and act on behalf of the users, manage the wireless resources in a decentralized fashion. Similar concepts have previously been proposed in [2]-[3] and they allow providers to share their infrastructure in an implicit fashion.

Just as user trade-agents compete for resources, revenue-seeking APs with overlapping coverage could, in an open access market, compete for wireless users (“traffic”). This could, for example, be achieved via occasional broadcasts of network status information to trade-agents in the proximity, who can use this to select the appropriate access.

The competition for resources among users, and the competition for users among APs, is here, viewed as two interrelated decision processes (games). In the paper we study how an open access market is likely to affect the users and network providers and the evaluation is based on game theory.

II. PRIOR WORK

In telecommunications, game theory has traditionally been used to design incentive-compatible pricing-schemes that increase the social value of a network shared by self-interested users.

Recently, however, network efficiency has been addressed from a provider-centric perspective [4]-[6] and the aim in these studies has been to devise dynamic pricing-schemes that internalize demand, quality-of-service, and resource consumption so that operator revenue can be maximized. A combined flow and admission control (AC) was designed in [4]. A related study was presented in [5] where a stochastic framework modelling the relationship between service-quality experienced by users, and their willingness to pay, was developed. Introducing dynamic pricing as an additional component in the AC was also the theme in [6]. Contrary to [4]-[5], a time-dynamic price-sensitive AC was used and this enabled operators to shape the traffic (“load balancing over time”). Apart from reducing the need for over-provisioning, this increased operator revenue.

Auctions are one way to implement time-dynamic demand-responsive pricing. There, aggregated demand determines the clearing price (“spot price”) for which the resource is sold. An early study that relied on auctions for partitioning network resources amongst competing users was [7] where the concept of “smart markets” was introduced. In the proposed architecture users informed the network how much they were willing to pay per transmitted packet. To allocate resources a secondary price sealed-bid auction (Vickrey auction) was utilized. A similar approach, although for a high speed downlink packet access (HSDPA) setting, was adopted in [8]. As in [7] the intention was to improve the resource efficiency and both approaches relied on trade-agents that resided in the network (APs). In [9], a divisible proportionally fair auction was used for allocating resources between competing users in a single-server system. Apart from developing a few agent strategies, e.g., minimize cost or service time, considerable effort was spent on guaranteeing that the strategies could be implemented in a decentralized manner. The study of proportionally fair
divisible auctions has later been extended in [10]-[11] where aspects such as user coalitions and revenue maximization for a single-server environment was addressed. In [12], we have previously extended decentralized agent-strategies from [9] to cover situations where the wireless resources from multiple APs, with varying service rates, are allocated with proportional division. The problem of selecting the reservation price ($\varepsilon$) that maximizes their expected revenue.

III. PROBLEM DEFINITION

Herein we study an agent-based architecture previously described in [12], where users can connect to multiple, potentially competing APs. To divide resources, i.e., downlink “air time” in a TDMA system between competing users, each AP employs a proportional fair divisible auction. In each auction all participating trade-agents obtain a share of the resource. This is in contrast to, e.g., a Vickrey auction where the highest bidder claims the entire resource.

Assuming that there are $N$ competing users, and letting $s_{i,j} \in [0, s_{\text{max}}]$ where $0 < s_{\text{max}} < \infty$ denote the bid user $j$ place in auction $i$ at AP $m$, the proportion of transmission time allocated to user $j$ is determined as

$$x_{i,j}^m = \frac{s_{i,j}^m}{\sum_{j=1}^N s_{i,j}^m + \varepsilon_m} = \frac{s_{i,j}^m}{s_{i,j}^m + s_{i,j-1}^m} = \frac{s_{i,j}^m}{p^m}$$

and we note that the allocation is proportionally fair by weight [9]. In (1) $\varepsilon_m \in (0, \varepsilon_{\text{max}}]$ is the reservation price, which is a nonzero bid placed by the auctioneer, located in the AP, that can be interpreted as a price floor below which the resource is not sold. Both $s_{i,j}^m$ and $\varepsilon_m$ are counted in monetary units (mu). It should be noticed that we envision that the APs will change their reservation price on a rather slow time-basis and they are here assumed to vary on the same time-scale as the average offered load. Faster variations are instead handled implicitly through the structure of the auction which, all else equal, results in higher prices when the AP is congested. Furthermore we assume that the resource is infinitesimally divisible.

To bid for resources users rely on selfish autonomous trade-agents whose aim is to maximize the expected utility (“value for money”) of its user. I.e., trade-agents do not account for how their decisions affect other users. The AP prioritization, which will be discussed further in Section IV, depends on the estimated average peak data-rate $R_{i,j}^m$ and the price for the resource $p^m = \sum_{j=1}^N s_{i,j}^m + \varepsilon_m$.

Given a deployment and the trade-agent strategies, the problem for the $n$th AP becomes to select a reservation price $\varepsilon_m$ so that its expected revenue is maximized (other instruments are achievable bit-rate, latency, etc.). A too low $\varepsilon_m$ may “attract” low-spending users and open up for user collusion, while a high-price policy may, on the other hand, dampen demand.

In this paper we study how revenue-seeking APs should select their reservation price in an environment where APs with overlapping coverage compete for users. Our aim is to establish whether AP competition can be self-sustained, and to what extent users could benefit from an open access market.

IV. USER MODEL

The development of a suitable bidding strategy for the agents originates from a specific service. In this study the modelled service is a file download, which is delay elastic. More specifically we say that the cost associated with a file transfer is dependent on the total time-duration, and the monetary expenditure, required to complete the download. Users’ desire for cheap and fast file transfers can be translated into a bidding strategy that describes how the trade-agent should act in different contexts. Throughout this paper we assume that agents utilize a strategy according to which they, in each individual auction, try to minimize the following cost function

$$c_{\text{auction}}^j = \frac{\sum_{m \in \mathcal{M}} s_{i,j}^m}{\sum_{m \in \mathcal{M}} \alpha_j x_{i,j}^m R_{i,j}^m} + \frac{\alpha_j}{\sum_{m \in \mathcal{M}} x_{i,j}^m R_{i,j}^m}.$$  (2)

Here $\mathcal{M}$ denotes the set of candidate APs, and $\alpha_j$ [mu/s] is a parameter that describes how sensitive a particular user is to delays. Minimization of (2) yields the best response (BR) function, or strategy, associated with trade-agent $j$ and it describes how the trade-agent should react to the actions of the other trade-agents. The latter can be summarized by the set $s_{-j} = \{s_{m,j}, \forall m \in \mathcal{M}\}$ and the BR function can, for trade-agent $j$, thus be written as

$$\varphi_j(s_{-j}) = \arg\min_{s_{m,j} \forall m \in \mathcal{M}} c_{\text{auction}}^m(s_{-j}).$$  (3)

A more detailed description of this strategy and the structure of the auction can be found in [12], but we note that the strategy converges to a unique, stable Nash equilibrium point (NEP). Notice the distinction between cost and monetary expenditure,
where the first refers to disutility (both delay and monetary expenditure) and the latter to the “cost” in monetary units.

As users enter and leave, say, AP $m$ the spot-price for which the resource is sold $\sum_{j=1}^{N} q_{i,j} + \varepsilon_m$ will vary and if the price is too high, a trade-agent is given the choice not to enter the system at present status. In this paper we assume that all users have an individual, and for the APs unknown, maximum price per MByte. If the expected monetary expenditure for transmitting one MByte, which can be estimated as

$$p = p(W) \approx E\left[\tilde{X}_{|\mathcal{M}|}\right] \frac{1}{W|\mathcal{M}|} \sum_{i=1}^{W} \sum_{m \in \mathcal{M}} \sum_{j} s_{i,j}^{m} + \varepsilon_m,$$  

(4)

exceeds this threshold the particular trade-agent will choose to refrain from transmission. In (4), $W$ is the number of auctions over which the average price is computed and $E[\tilde{X}_{|\mathcal{M}|}]$ [s/MByte] corresponds to the expected transmission time that a randomly selected user, served by $|\mathcal{M}|$ APs, needs in order to transfer one MByte. The latter can approximated by

$$E[\tilde{X}_{|\mathcal{M}|}] \approx 8 \int_{R_{\text{min}}}^{R_{\text{max}}} \frac{f_{R_{|\mathcal{M}|}}(R)}{R} dR \quad [\text{s/MByte}]$$  

(5)

where $f_{R_{|\mathcal{M}|}}(R)$ is the probability density function of the peak data-rate [Mbit/s] for a user with $|\mathcal{M}|$-branch selection diversity. Here we assume that the APs broadcast information about the price per MByte on a per-auction basis.

Together users’ price thresholds form the acceptance probability function $P_A(p)$ that describes the probability with which a randomly chosen trade-agent will commence a file transfer as a function of $p$. Utilizing the acceptance probability, the aggregated user-demand can be written as

$$D(p) = D_0 P_A(p) \quad [\text{Mbit/s}]$$  

(6)

where $D_0$ is the potentially offered load and we assume that the aggregated demand function is perfectly known for all APs. The demand consists of files, with an expected size $q$ [Mbit] that arrive to the system of APs according to a Poisson process characterized by an intensity $\lambda$ and consequently $D_0 = q\lambda$.

Three demand functions are considered; one concave, one linear, and one convex (see Table I) and for comparison reasons they have all been normalized so that

$$\int_{0}^{\infty} P_A(p) dp = \frac{1}{\beta},$$  

(7)

where $\beta$ is demand-related parameter. This ensures that an AP able to apply first order price discrimination will earn the same revenue, irrespectively of which one of the three demand functions that is used [13].

The linear demand function depicts a scenario where the users’ price thresholds are uniformly distributed between 0 and $\frac{q}{2}$. The concave and convex demand functions instead represent skew price distributions. With the convex function the majority of users are “price-sensitive” and consequently have a relatively low threshold. Some users, however, are very insensitive to price and the difference amongst users can thus be considerable. For the concave demand function, finally, users are centered around one threshold and the difference between the users is smaller than both the linear and convex case. Figure 2 presents the acceptance probability functions (“normalized” demand functions) as well as the associated price elasticity of demand. The latter is defined as

$$E_D(p) = \left|\frac{\partial D(p)}{\partial p}\right| \frac{p}{D(p)} = \frac{\% \text{ change in quantity}}{\% \text{ change in price}},$$  

(8)

and it measures the rate with which the quantity demanded varies due to a price change. If $E_D(p) < 1$ the demand is said to be price inelastic whereas if $E_D(p) > 1$ it is instead referred to as price elastic. The special case where $E_D(p) = 1$ corresponds to the price that a revenue-maximizing AP would utilize in a monopoly situation [13].

Fig. 2. Graphical representation of the studied acceptance probability $P_A$ (normalized demand) functions and their price elasticity of demand $E_D$.

V. THE AP PROBLEM – REVENUE MAXIMIZATION

As we already have touched upon, we model the open access market as two interconnected games. The first game is between the users, which try to minimize their expected cost for transferring files, while the individual APs form the set of players in the second game. Just as user trade-agents want to minimize their cost function, APs try to maximize their expected revenue per second $\lambda_m(\cdot) \ [\mu$/s]. As mentioned already in Section III, this, for the APs, translates into selecting an appropriate reservation price $\varepsilon_m$. The domain of $\varepsilon_m$, thus, forms the strategy space for each AP.

Although the maximization of the expected revenue is straightforward in scenarios where the involved APs cooperate,

<table>
<thead>
<tr>
<th>User demand</th>
<th>Price elasticity</th>
<th>Shape of demand</th>
</tr>
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<tbody>
<tr>
<td>$q\lambda e^{-\beta p}$</td>
<td>$\beta p$</td>
<td>“concave”</td>
</tr>
<tr>
<td>$q\lambda (1 - \frac{p}{2})$</td>
<td>$\beta p$</td>
<td>“linear”</td>
</tr>
<tr>
<td>$q\lambda (1 - \frac{(\alpha p)^2}{2})$</td>
<td>$(-\alpha p)^2$</td>
<td>“concave”</td>
</tr>
</tbody>
</table>

TABLE I

SUMMARY OF EMPLOYED DEMAND FUNCTIONS.
which we model by the Nash bargaining solution (NBS), finding the reservation price in contexts where APs compete is rather cumbersome task. In the latter case APs are involved in a noncooperative game. This section describes, and analyzes, the problem at hand for the APs in scenarios where they

A. Compete with each other for the same wireless users,
B. Cooperate and try to maximize their total revenue, or
C. Divide the users so that a fraction $\frac{1}{|M|}$ is assigned to each of the $|M|$ AP.

A. Competing Access Points
Let $\Lambda_m(\varepsilon_m, \varepsilon_{-m})$ denote the expected revenue per second that AP $m$ earns if it use a reservation price $\varepsilon_m$ at the same time as the other APs utilize reservation prices $\varepsilon_{-m} = \{\varepsilon_i, \forall i \in M \setminus m\}$. Given the actions of the other APs the problem faced by AP $m$ can be formulated as

$$d_m(\varepsilon_{-m}) = \arg \max_{\varepsilon_m} \Lambda_m(\varepsilon_m, \varepsilon_{-m}).$$ (9)

Here $d_m(\varepsilon_{-m})$ is the BR function associated with AP $m$. The NEP

$$\varepsilon^*_m = d_m(\varepsilon^*_{-m}) \forall m \in M,$$ (10)

forms the solution to the noncooperative AP game. The NEP is characterized by that no AP can, unilaterally, increase its expected revenue by deviating from its current reservation price. Figure 3 shows the BR functions for an AP game where users are described by the convex demand function and the potentially offered load $D_0 = 3.2$ Mbit/s/AP ($\lambda = 0.4$ files/s/AP). As all APs are assumed to be identical (system bandwidth, transmit power, etc.) we confine ourselves to symmetric solutions where $\varepsilon^*_k = \varepsilon^*_l$, $\forall k,l \in M$. Due to the interrelation between the user game and the AP game we cannot present a proof of the existence or the stability of the NEP of the noncooperative AP game. However, the simulation experiments, one example of such in Figure 3, gives strong indications that a stable unique NEP does exist.

B. Cooperative Access Market
For comparison, we also study the case where APs cooperate. We model this as a Nash bargaining game where the NEP associated with the noncooperative AP game is used as disagreement point. Given these assumptions, the problem can be written as

$$\max_\varepsilon \sum_{m \in M} \Lambda_m(\varepsilon) \text{ where } \varepsilon_m = \varepsilon, \forall m \in M.$$ (11)

By combining (6), (8) and (11) we can furthermore derive the optimal reservation price as

$$\varepsilon^* = \frac{E_D^{-1}(1)}{E(\tilde{X}_M)} \sum_{m \in M} \sum_{j=1}^N s^m_j$$ (12)

where $E_D^{-1}(1)$ is the average price per MByte corresponding to a price elasticity of demand that is equal to one. By further noticing that the expected revenue per second, and AP, can be written as $\tilde{\Lambda} = \sum_{m \in M} \Lambda_m = D(p)p$ we may write the revenue, that corresponds to the NBS, cooperating APs can earn, $\tilde{\Lambda}^*$, as

$$\tilde{\Lambda}^* = D(\tilde{E}_D^{-1}(1)) \cdot E_D^{-1}(1).$$ (13)

Figure 4 presents the average revenue per second and AP as a function of the reservation price $\varepsilon$ in a situation where on average 0.8 files/s enters the systems. Notice that for this example APs can earn approximately 50 percent higher revenue by cooperating.

C. Market Split
In the third and final scenario, APs simply divide users so that a fraction $\frac{1}{|M|}$ is served by AP $m$. This case can be treated in a similar manner as the cooperative case.

VI. NUMERICAL EXAMPLE
This section applies the developed framework to an example with two APs that either compete, cooperate, or divide the wireless users between them. Files of fixed size $q$ arrive
would experience if APs cooperated and utilized monopoly pricing. According to a Poisson process and resources are allocated, once per second, according to the NEP associated with the user game described in Section IV. To quantify the performance experienced by users we utilize the average user throughput and monetary expenditure per Mbyte. For APs we, instead, use the average revenue per second. For each studied scenario, and arrival intensity $\lambda$, a time-duration equal to 40 hours (with a granularity of one second) have been simulated and it should be noticed that all presented results has been evaluated at equilibrium of the respective game.

The pathloss has been modelled as

$$L(r) = 35.3 + 38\log_{10}(r),$$

where $r$ [m] denotes the distance between an AP and terminal. Throughout the simulations we have neglected shadow fading, modelled the interference as coming from constantly transmitting neighboring APs, and utilized a truncated version of the Shannon bound (adjusted for efficiency losses) where the peak data-rates are given as

$$R_{i,j} = \min\left(R_{\text{max}}, W \log_2 \left(1 + \frac{\Gamma_{i,j}}{2}\right)\right).$$

Here $W=3.84$ MHz is the effective channel bandwidth and $\Gamma_{i,j}$ is the signal-to-interference-plus-noise ratio (SINR) that user $j$ experience in auction $i$. Notice that we also, for simplicity reasons assume that the peak data-rates are uncorrelated, both between users, and APs. Remaining simulation parameters are summarized in Table II.

**A. User Performance**

Figure 5 and Figure 6 present the average price per MByte and throughput experienced by the users in the studied scenarios. As comparison we have included the performance that users would experience if APs cooperated and utilized monopoly pricing (notice that the values for the "convex" and "linear" demand functions coincide for the cooperating case).

As expected, we see that an architecture where APs compete and share their resources implicitly, in combination with user-employed trade-agents, has the potential to reduce price, as well as increase throughput significantly. Notice that neither price, nor the experienced throughput depend on the potential offered load $D_0$ for the case where APs cooperate. Although nonintuitive at first this is in alignment with the analysis presented in Section V-B where we showed that the optimal price for a revenue-maximizing AP was $E_D^{-1}(1)$ (monopoly price). Depending on the shape of the demand function the monopoly price per MByte varies; see Figure 5, which also leads to a slight change in the experienced user throughput. However, a slightly larger difference can be seen in the throughput if we compare APs that cooperate or split the market between them. APs that have divided the users among them (market splitting APs) will lose in terms of macroscopic diversity, and are thus not able to deliver the same throughput as cooperating APs, see Figure 6. Note that the we have omitted the results from the simulations for the monopoly cases, and rely on validated analytical approximations.

Comparing the cooperative and competitive scenario, we see that the difference in user performance is largest with convex demand, and smallest for the concave function. This can be

**TABLE II**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tbody>
<tr>
<td>Relative delay sensitivity ($\alpha$)</td>
<td>0.1</td>
</tr>
<tr>
<td>AP transmit power [W]</td>
<td>20</td>
</tr>
<tr>
<td>Cell radius [m]</td>
<td>440</td>
</tr>
<tr>
<td># coexisting APs ($</td>
<td>\mathcal{M}</td>
</tr>
<tr>
<td>File size ($q$) [Mbyte]</td>
<td>1</td>
</tr>
<tr>
<td>Demand constant ($\beta$)</td>
<td>0.0066</td>
</tr>
<tr>
<td>Price averaging window ($W$) [s]</td>
<td>300</td>
</tr>
<tr>
<td>Maximum bit-rate ($R_{\text{max}}$) [Mbit/s]</td>
<td>7</td>
</tr>
</tbody>
</table>

**SUMMARY OF THE UTILIZED SIMULATION PARAMETERS**
We have developed a framework based on game-theory for studying demand-responsive radio resource management (RRM) in contexts where multiple access points (APs), with overlapping coverage, compete for the opportunity to serve users. To prioritize between candidate APs users rely on autonomous terminal based trade-agents, that act on behalf of the users, and herein they base their access selection and resource consumption decisions on the price and peak data-rates so that their utility (“value for money”) is maximized. In similar manner, APs try to control the price that maximize their individual revenue. Together this offers an market-based solution for allocating resources as well as users to the different APs. The framework was used to analyze a scenario with two overlapping APs that either cooperated, and shared the resources explicitly, or competed. In the latter the APs were shared implicitly.

Our results indicate that establishing a competitive access market where the infrastructure is shared implicitly can improve the utility experienced by users considerably compared to a scenario were APs cooperate and try to maximize their aggregated revenue. Largest gains were observed when the demand function is convex whereas the smallest improvement occurred with concave demand. Although AP competition reduces AP revenue, this loss may, depending on the shape of demand, be as small as 10% (for reasonable loads). As the employed demand model furthermore is oblivious to the “quality-of-service” (throughput) experienced by users, this loss is most likely overestimated.

Fig. 7. Relative revenue gain for cooperating APs as a function of λ. We see that although APs always will benefit from cooperation, the relative gains decrease rather quickly with λ. This means that as long as systems not are overprovisioned (and resource management is a problem) an architecture where APs compete for the users is feasible.

explained by the price elasticity $E_D$ presented in Figure 2. Recall from Section III that the more inelastic demand is to price changes, the smaller $E_D$ becomes. Hence an inelastic demand generally results in higher prices in situations where APs compete. In fact we may observe; see Figure 2; that the concave demand function has smallest $E_D$ (for all prices below 120 μu/MByte). Similarly the convex function, which resulted in lowest cost, is associated with the highest price elasticity of demand. We furthermore recall that the price was determined by the total demand and typically increased monotonously with the number of users in the system. Hence, a high price can be interpreted as an indication of that “many” trade-agents have to share the available resources and consequently we would expect that the throughput experienced by users, who entered the system, becomes smaller for concave demand functions.

B. Network Performance

Figure 7 presents the relative revenue that the APs would gain if they choose to cooperate instead of compete for users. As expected we see that APs that compete earn less than those who cooperate. However, as the potentially offered load increases the difference between cooperation and competition decreases. For the concave demand function, for example, the difference is marginal at loads exceeding 3.2 Mbit/AP/s. In general we see that the loss in revenue is less pronounced than the quality improvement for the users (see Figure 5 and Figure 6). This can be explained by that competition between APs results in a price lower than the monopoly price, which in turn results in that a larger fraction of the users enter the system.

VII. CONCLUSIONS

We have developed a framework based on game-theory for studying demand-responsive radio resource management.