A Distributed Dynamic Target-SIR Tracking Power Control Algorithm for Wireless Cellular Networks

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Abstract—The well-known fixed target-SIR tracking power control algorithm (TPC) provides all users with their given feasible fixed target-SIRs, but cannot improve the system throughput even if additional resources are available. The opportunistic power control algorithm (OPC) significantly improves the system throughput but cannot guarantee the minimum acceptable signal-to-interference-ratio (SIR) for all users (unfairness). To optimize the system throughput subject to a given lower bound for the users’ SIRs, we present a distributed dynamic target-SIR tracking power control algorithm (DTPC) for wireless cellular networks by using TPC and OPC in a selective manner. In the proposed DTPC, when the effective interference (the ratio of the received interference to the path-gain) is less than a given threshold for a given user, that user opportunistically sets its target-SIR (which is a decreasing function of the effective interference) to a value higher than its minimum acceptable target-SIR; otherwise it keeps its target-SIR fixed at its minimum acceptable level. We show that the proposed algorithm converges to a unique fixed point starting from any initial transmit power level in both synchronous and asynchronous power updating cases. We also show that our proposed algorithm not only guarantees the (feasible) minimum acceptable target-SIRs for all users (in contrast to OPC), but also significantly improves the system throughput as compared to TPC. Furthermore, we demonstrate that DTPC along with TPC and OPC can be utilized to apply different priorities of transmission and service requirements among users. Finally, when users are selfish, we provide a game theoretic analysis of our DTPC algorithm via a non-cooperative power control game with a new pricing function.

Index Terms—Distributed power control, dynamic target-SIR allocation, wireless cellular networks.

I. INTRODUCTION

Power control plays an important role in satisfying the increasing demand for high data rates in wireless data networks. Enhancing the throughput of a user or of the system is highly dependent on how interference is managed. The objective for power control from a user’s point of view is to support a user with its minimum acceptable throughput; whereas from a system’s point of view is to maximize the aggregate throughput. These two objectives are completely opposed to one another, since in the former, it is required to compensate for the near-far effect by allocating higher power levels to users with poor channels as compared to users with good channels, and in the latter, high power levels are allocated to a few users with best channels and very low (even zero) power levels to others.

A distributed power control is practically preferred to a centralized one, because in the former, the transmit power level of a user is decided by the user utilizing the local information and minimal feedback from the base station. In contrast, a centralized approach needs to have path-gains and throughput requirements for all users at the base station. To satisfy the two stated objectives for power control, two distributed approaches have been considered, namely the fixed-target-SIR-tracking proposed in [1], and the opportunistic approaches proposed in [2]–[4].

In the fixed-target-SIR-tracking power control algorithm (TPC), each user is tracking its own predefined fixed target-SIR. The TPC that was proposed in [1] enables users to achieve their fixed target-SIRs at minimal aggregate transmit power if the target-SIRs are feasible. However, there is a major drawback in the original TPC [5]. It causes users to exactly hit their fixed target-SIRs in feasible systems even if additional resources are still available that can otherwise be used to achieve higher SIRs (and thus better throughputs). Besides, the fixed-target-SIR assignment is suitable only for voice service for which reaching a SIR value higher than the given target value has no practical effect on service quality (due to characteristics of the service and human ears). In contrast, for data services, a higher SIR results in a better throughput, which is desirable. Thus, it is important to design a power control algorithm for wireless data networks by which the minimum acceptable target-SIRs (which are assumed to be feasible) are guaranteed for all users, and at the same time, the system throughput is increased to the extent that the required resources are available by increasing the actual SIRs received by some users.

From the system’s point of view, OPC allocates high power levels to users with good channels (experiencing high path-gains and low interference levels), and very low power to users with poor channels. In this algorithm, a small difference in path-gains between two users may lead to a large difference in their actual throughputs [2], [3]. Since an opportunistic algorithm always favors those users with better channels, it magnifies unfairness. For users with low-mobility (when their
channels vary slowly or are static), this might lead to long-term unfairness.

The characteristics of existing distributed power control schemes are summarized as follows. TPC can provide all users with their fixed target-SIRs when the system is feasible, but cannot improve the system throughput further even if additional resources are available. OPC significantly improves the system throughput but cannot guarantee the minimum acceptable SIRs for some users (unfairness).

Motivated by the above drawbacks, in this paper, we formally define the problem of system throughput maximization subject to a given feasible lower bound for the minimum SIRs of all users in wireless cellular networks; and propose a distributed dynamic power control algorithm to address this problem. We will show that when the minimum acceptable target-SIRs are feasible, the actual SIRs received by some users can be dynamically increased (to a value higher than their minimum acceptable target-SIRs) in a distributed manner, so far as the required resources are available and the system remains feasible (meaning that reaching the minimum target-SIRs for the remaining users are guaranteed). This would enhance the system throughput (at the cost of higher power consumption) as compared to TPC. We will show that our proposed algorithm not only guarantees the (feasible) minimum acceptable target-SIRs for all users as is the case by TPC (and in contrast to OPC), but also significantly improves the system throughput as compared to TPC. Furthermore, we discuss the application of DTPC to cognitive radio networks and multi-service networks; and provide a game theoretic analysis of our proposed algorithm.

The rest of this paper is organized as follows. In Section II, we introduce the system model and throughput measure. In Section III, we review the existing distributed power control algorithms, present a formal statement of the problem, and state the objectives. Section IV contains the proposed method, and an analysis of its convergence and its improved system throughput. Application of our proposed algorithm in cognitive radio or multi-service networks is discussed in Section V. Section VI provides a game theoretic analysis of our proposed algorithm. Simulation results and conclusions are presented in Sections VII and VIII, respectively.

II. SYSTEM MODEL AND SIR FEASIBILITY

We consider a multi-cell wireless CDMA network with $K$ base stations (cells) and $M$ active users denoted by $\mathcal{K} = \{1, 2, ..., K\}$ and $\mathcal{M} = \{1, 2, ..., M\}$, respectively. Let $p_i$ be the transmit power of user $i$. Noise is assumed to be additive white Gaussian whose power at the receiver of the base station $k$ is $\sigma_k^2$. Let $s_i$ denote the base station to which the user $i$ is assigned. Denote the set of users assigned to the base station $k$ by $C_k = \{i \in \mathcal{M} | s_i = k\}$.

The receiver is assumed to be a conventional matched filter. Thus, for a given transmit power vector $\mathbf{p} = [p_1, p_2, ..., p_M]^T$, the SIR of a user $i$, denoted by $\gamma_i$ is

$$
\gamma_i(\mathbf{p}) = \frac{g_i}{h_{s_i,k}} \frac{p_i}{I_i(\mathbf{p})},
$$

where $g_i$ is the processing gain for user $i$ (defined as the ratio of chip rate (or the spreading bandwidth) to the transmit data rate), $h_{s_i,k}$ is the path-gain from user $i$ to its assigned base station and $I_i(\mathbf{p}) = \sum_j h_{s_j,k}^j + \sigma_k^2$ is the interference experienced by user $i$ at its assigned base-station. The interference $I_i(\mathbf{p})$ can be rewritten as $I_i(\mathbf{p}) = I_{i}^{\text{int}} + I_{i}^{\text{ext}} + \sigma_k^2$, where $I_{i}^{\text{int}} = \sum_{j \in \mathcal{C}_k, j \neq i} p_j h_{s_j,k}^j$ and $I_{i}^{\text{ext}} = \sum_{j \notin \mathcal{C}_k} p_j h_{s_j,k}^j$ are the intra-cell interference and the inter-cell interference, respectively. The effective interference for user $i$ is defined as the ratio of its experienced interference to its path-gain [3], denoted by $R_i$.

$$
R_i(\mathbf{p}) = \frac{I_i(\mathbf{p})}{g_i h_{s_i,k}^i}.
$$

The value of $R_i$ represents the channel status for user $i$, i.e. for a given processing gain, a higher interference and a lower path-gain result in a higher $R_i$, implying a poor channel as compared to a lower interference and a higher path-gain which result in a lower $R_i$, implying a good channel.

Using matrix notations, the relation between the transmit power vector and the SIR vector can be rewritten as

$$
\mathbf{p} = \mathbf{G} \mathbf{p} + \mathbf{\eta}
$$

where the $(i,j)$ component of $\mathbf{G}$ is $G_{i,j} = (h_{s_j,k} \gamma_i)/(g_i h_{s_i,k}^i)$ if $i \neq j$, and $G_{i,i} = 0$ if $i = j$, and the $(i,j)$ component of $\mathbf{\eta}$ is $\eta_i = (\sigma_k^2 \gamma_i)/(g_i h_{s_i,k}^i)$. A SIR vector $\gamma$ is feasible if a power vector $\mathbf{p} \geq 0$ exists that satisfies (3). It was shown in [6] that the necessary and sufficient condition for feasibility of a given SIR vector $\gamma$ is $\rho(\mathbf{G}) < 1$, where $\rho(\mathbf{G})$ is the spectral radius (maximum eigenvalue) of the matrix $\mathbf{G}$.

From (3) and with some mathematical manipulations, a one-to-one relation between a transmit power vector and the achievable SIR vector is obtained [4], [7], [8], that is

$$
p_i = \frac{\gamma_i}{h_{s_i,k}(\gamma_i + g_i)} \times \frac{\sigma_k^2 + I_{i}^{\text{ext}}}{1 - \sum_{j \notin \mathcal{C}_k} \frac{\gamma_j}{\gamma_j + g_j}}, \quad \text{for all } i \in \mathcal{M}.
$$

Thus a SIR vector is feasible if

$$
\sum_{j \in \mathcal{C}_k} \frac{\gamma_j}{\gamma_j + g_j} < 1, \quad \text{for all } k \in \mathcal{K}.
$$

The interesting and useful point from the feasibility constraint (5) is that feasibility can be individually checked in each cell by checking the sum of $\gamma_j/(\gamma_j + g_j)$ for all users in that cell. Note that for checking the feasibility, using (5) is simpler, as compared to checking $\rho(\mathbf{G}) < 1$ that requires calculation of the spectral radius of the matrix $\mathbf{G}$.

The left-side of the inequality (5) is the Pareto efficiency measure for users’ SIR allocations in the sense that for a given base-station $k$, as the sum of $\gamma_j/(\gamma_j + g_j)$ approaches 1, the SIR of no user assigned to that base-station (i.e. $C_k$) can be further increased without decreasing the SIRs of some other users in $C_k$. In other words, a value of $\sum_{j \in \mathcal{C}_k} \frac{\gamma_j}{\gamma_j + g_j}$ closer to 1 for each $k \in \mathcal{K}$ means a more Pareto efficient use of resources and a higher system throughput; and a value farther from 1 means more resources are available to increase the users’ SIRs.
Similar to [4], [7], [9] and [10], we use an information theoretic approach to define the throughput for each user \(i\) by

\[ T_i(p) = W \log_2(1 + \gamma_i(p)) , \]

where \(W\) is the channel bandwidth. The system throughput (the sum of achievable rates by users) is

\[ T(p) = \sum_i T_i(p) . \]

III. Problem Formulation and Background

A. Existing Distributed Power Control Algorithms

In a distributed power control algorithm, each user \(i\) updates its transmit power by a power-updating function \(f_i(p)\), i.e.

\[ p_{i}(t+1) = f_i(p(t)) \]

where \(p(t)\) is the transmit power vector at time \(t\). The fixed-point of the power-updating function, denoted by \(p^*\), is obtained by solving \(p^* = f(p^*)\). If a distributed power control algorithm converges to an equilibrium state, it will be a fixed-point of the corresponding power-updating function.

The power-updating function of TPC is

\[ f_i^{(T)}(p(t)) = \hat{\gamma}_i R_i(p(t)) \]

where \(\hat{\gamma}_i\) is the given target-SIR for user \(i\). It was shown in [1] and [11] that if the target-SIR vector \(\hat{\gamma}\) is feasible, then TPC converges either synchronously or asynchronously to a fixed point at which users attain their target-SIRs with minimum aggregate transmit power, i.e. its fixed point solves the following optimization problem

\[ \min_{p \geq 0} \sum_i p_i \]

subject to \(\gamma_i(p) \geq \hat{\gamma}_i, \forall i \in \mathcal{M} \).

In OPC, the transmit power levels are updated in a manner opposite to TPC, i.e. it is increased when the channel is good and is decreased when the channel is poor. The power-updating function of OPC is

\[ f_i^{(O)}(p(t)) = \frac{\eta_i}{R_i(p(t))} \]

where \(\eta_i\) is a constant for user \(i\). In this algorithm, each user \(i\) tries to keep the product of its transmit power and its effective interference to a constant \(\eta_i\), called the target signal-interference product (SIP). This algorithm is convergent, and although it does not guarantee optimal system throughput defined by

\[ \max_{p \geq 0} \sum_i T_i(p) , \]

it significantly enhances the system throughput by transmitting at high power levels by users with good channels, and transmitting at low power levels by users with poor channels; but leads to unfairness as well. In TPC and OPC, each user sets its own target-SIR and its own target-SIP, respectively, meaning that each user requires only to know its SIR measured at the base station (i.e. minimal feedback information). Note that each user \(i\) can obtain the value of \(R_i(p(t))\) in (8) and (10) by knowing its achieved SIR at its assigned base-station by using \(R_i(p(t)) = p_i(t)/\gamma_i(p(t))\).

B. Problem Statement and Objectives

We now introduce the problem of system throughput optimization subject to a given feasible lower bound for users' SIRs as

\[ \max_{p \geq 0} \sum_i T_i(p) \]

subject to \(\gamma_i(p) \geq \hat{\gamma}_i, \forall i \in \mathcal{M} \).

The constraint on the received SIR, i.e. \(\gamma_i(p) \geq \hat{\gamma}_i\), is equivalent to a constraint on throughput, i.e. \(T_i(p) \geq \hat{T}_i\), where \(\hat{T}_i = W \log_2(1 + \hat{\gamma}_i)\). It can be shown that the above optimization problems is non-convex and thus it cannot be solved by using the conventional methods. Our focus in this paper is to design a distributed convergent power control algorithm that can significantly improve the system throughput while guaranteeing the minimum acceptable target-SIRs for all users.

IV. The Proposed Method: Dynamic Allocation of Target-SIRs

When the minimum acceptable target-SIRs are feasible, the actual target-SIRs tracked by some users with better channels should be dynamically set to a value higher than their minimum acceptable target-SIRs to the extent that the required resources are available and the system remains feasible (i.e. the constraint (5) is satisfied). This will enhance the system throughput (at the cost of consuming more power).

It is well-known that for enhancing the system throughput, users with good channels should transmit at higher power levels compared to other users [2], [3]. On the other hand, reducing the outage necessitates that users with poor channels also transmit at just enough power to at least reach their minimum acceptable SIRs. Based on these observations, we propose that users with good channels set their power levels in an opportunistic manner and users with poor channels transmit in a minimum-acceptable-target-SIR tracking manner. This can be done in a distributed manner if each user \(i\) updates its transmit power according to the following power control algorithm

\[ f_i^{(D)}(p(t)) = \begin{cases} \eta_i & \text{if } R_i(p(t)) < R_i^{th} \\ \hat{\gamma}_i R_i(p(t)), & \text{if } R_i(p(t)) \geq R_i^{th} \end{cases} \]

where \(R_i^{th}\) is the effective interference threshold, \(\eta_i\) is a constant, and \(\hat{\gamma}_i\) is the minimum-acceptable-target-SIR for user \(i\). Note that this algorithm is a generalized selective scheme of either OPC or TPC. At the extreme cases where \(R_i^{th} \to 0\) or \(R_i^{th} \to \infty\), the proposed power-updating function turns into either TPC or OPC, respectively.

Our proposed dynamic target-SIR tracking power control algorithm (DTCP) is explained below. We rewrite (13) as the DTCP power-updating function

\[ f_i^{(D)}(p(t)) = \hat{\gamma}_i R_i(p(t)) \]

in which \(\hat{\gamma}_i(p(t))\) is a target-SIR dynamically set as

\[ \hat{\gamma}_i(p(t)) = \begin{cases} \eta_i & \text{if } R_i(p(t)) < R_i^{th} \\ \gamma_i, & \text{if } R_i(p(t)) \geq R_i^{th}. \end{cases} \]
For DTPC to be continuous, its three parameters are adjusted as follows. For a given $\tilde{\gamma}_i$ and $\eta_i$, the value of $R_i^{th}$ is set as

$$R_i^{th} = \sqrt{\frac{\eta_i}{\tilde{\gamma}_i}}. \quad (16)$$

The minimum acceptable target-SIR in (14) is set exactly to the same value of the fixed-target-SIR that was set in (8). We use fixed target-SIR and minimum acceptable target-SIR interchangeably throughout this paper. As seen in Fig. 1, by using DTPC, a given user sets its target-SIR at the minimum acceptable value when the channel is not good (the value of the effective interference is higher than the threshold). When its channel is good, it opportunistically sets its target-SIR at a value higher than the minimum acceptable value. When the set of minimum target-SIRs are feasible in a non-Pareto efficient manner (i.e. additional resources are available that can be used to further enhance users’ SIRs, meaning that the sum of $\frac{\gamma_j}{\gamma_i + \gamma_j}$ is far from 1), some users with better channels reach higher SIRs as compared to the minimum acceptable value, and the remaining users actually hit their minimum target-SIRs. This means that the system throughput is increased while the users’ minimum target-SIRs are guaranteed, as we will show in the sequel. Such enhancement on the system throughput causes all users to consume more power as compared to the fixed (minimum-acceptable) target-SIR tracking scheme.

### A. Convergence of DTPC

A framework for examining the convergence of the so called standard power-updating functions such as TPC was provided in [11]. This framework was generalized to a new framework in [2], applicable to a wider range of distributed power control algorithms (covering both TPC and OPC) with a two-sided scalable power updating function. Our proposed DTPC falls into this generalized framework. Using the key properties of a two-sided scalable function, we will show that the DTPC’s power-updating function has a unique fixed point to which it converges. To show this, we define the two-sided scalable functions as in [2], and prove that the DTPC’s power-updating function (14) is two-sided scalable.

### Definition 1:
A power-updating function $f(p) = [f_1(p), f_2(p), ..., f_M(p)]^T$ is two-sided scalable if for all $a > 1$, $\frac{1}{a} p \leq p' \leq a p$ implies

$$\frac{1}{a} f_i(p) \leq f_i(p') \leq a f_i(p) \text{ for all } i \in M. \quad (17)$$

### Theorem 1:
The DTPC’s power-updating function $f^{(D)}(p)$ in (14) is two-sided scalable.

**Proof:** It has been shown in [2] that for the given two sided scalable functions $f(p)$ and $f'(p)$, their component-wise maximum or minimum, i.e. $\max\{f(p), f'(p)\}$ or $\min\{f(p), f'(p)\}$, respectively, are two-sided scalable. Since the power-updating functions corresponding to the fixed-target-SIR tracking and the opportunistic power control algorithms are two sided-scalable [2], and since the proposed power-updating function can be rewritten as $f^{(D)}_i(p(t)) = \max\left\{ \frac{\eta_i}{R_i(p(t))} \tilde{\gamma}_i R_i(p(t)) \right\}$, then this theorem is proved.

### Theorem 2:
If there exists a fixed point for the proposed power-updating function, i.e. if there exists a $p^{\ast(D)}$ so that $p^{\ast(D)} = f(p^{\ast(D)})$, then

I. This fixed point is unique, and
II. For any initial power vector, the power control algorithm $p(t + 1) = f(p(t))$ converges to this unique fixed-point.

**Proof:** This theorem is directly concluded from Theorem 1 as a key property of a two-sided scalable function [2].

### Theorem 3:
There exists a fixed-point for the DTPC’s power-updating function $f^{(D)}(p)$ if the minimum acceptable target-SIR vector $\tilde{\gamma}$ is feasible.

**Proof:** It has been proved in [2] that if $f(p)$ is two-sided scalable, continuous, and $f(p) \leq u$ for all $p$ (i.e. $f(p)$ is upper bounded) for $u > 0$, then a fixed point for $f$ exists. Since the proposed power-updating function is two-sided scalable and continuous, by employing (16), the existence of a fixed-point for the proposed power-updating function is guaranteed if it is upper bounded. Now we show that when $\tilde{\gamma}$ is feasible, the DTPC’s power-updating function is upper bounded. When $\tilde{\gamma}$ is feasible, a fixed-point denoted by $p^{\ast(T)}$ exists for the power-updating function of TPC (8). If $R_i(p^{\ast(T)}) \geq R_i^{th}$ for all $i \in M$, then $p^{\ast(T)}$ is also the fixed-point of the proposed power-updating function for DTPC denoted by $p^{\ast(D)}$, i.e. $p^{\ast(D)} = p^{\ast(T)}$. This means that at the fixed-point of DTPC, all users are in the TPC mode. Otherwise, at the fixed-point of DTPC, there exist some users (at least one user) operating in the OPC mode, i.e. for which $R_i(p^{\ast(D)}) < R_i^{th}$ and the remaining users are operating in the TPC mode. For TPC, the existence of a fixed-point depends only on path-gains and target-SIRs, and does not depend on the level of background noise [12]. On the other hand, the transmit power of users operating in the OPC mode appear as the background noise to users operating in the TPC mode. The transmit power levels set by users in the OPC mode are upper bounded because the OPC’s power-updating function is always upper bounded, and the transmit power levels set by users operating in the TPC mode are also upper bounded because the set of their fixed-target-SIRs is feasible.
B. Improvement in System Throughput by Using DTPC

In the following theorem, we will show that by using DTPC for a given feasible set of minimum target-SIRs, the system throughput is improved as compared to TPC, while guaranteeing that all users are supported at least with their minimum target-SIRs.

**Theorem 4:** Given a feasible target-SIR vector \( \widehat{\gamma}_i \), let \( p^{*}(T) \) and \( p^{*}(D) \) be the fixed points of TPC and DTPC, respectively. We have \( \gamma_i(p^{*}(D)) \geq \widehat{\gamma}_i \) for all \( i \in M \) and thus \( T(p^{*}(D)) \geq T(p^{*}(T)) \). More specifically,

I. If and only if \( R_i(p^{*}(T)) \geq R_i^{th} \) for all \( i \), then \( \gamma_i(p^{*}(D)) = \widehat{\gamma}_i \) for all \( i \in M \) and thus \( T(p^{*}(D)) = T(p^{*}(T)) \).

II. If and only if there exists at least one user with \( R_i(p^{*}(T)) < R_i^{th} \), then for a non-empty set of users, we have \( \gamma_i(p^{*}(D)) > \widehat{\gamma}_i \), and for the remaining users we have \( \gamma_i(p^{*}(D)) = \widehat{\gamma}_i \), and thus \( T(p^{*}(D)) > T(p^{*}(T)) \).

**Proof:** If \( R_i(p^{*}(D)) \geq R_i^{th} \) for all \( i \), then from (8) and (14), it is easy to see that the fixed-point of TPC is the same as the fixed-point of DTPC, i.e. \( p^{*}(D) = p^{*}(T) \). Thus, in this case, we have \( \gamma_i(p^{*}(D)) = \gamma_i(p^{*}(T)) \) for all \( i \in M \), which proves Part I. For Part II, the (non-empty) set of users with \( R_i(p^{*}(T)) < R_i^{th} \) is denoted by \( N \neq \emptyset \). In this case, there exists a non-empty subset of users in \( N \), denoted by \( L \), for which we have \( \Gamma_i(p^{*}(D)) > \widehat{\gamma}_i \), i.e. \( R_i(p^{*}(D)) < R_i^{th} \) for all \( i \in L \). If this is not true, then we have \( R_i(p^{*}(D)) \geq R_i^{th} \) for all \( i \in M \), meaning that \( p^{*}(D) = p^{*}(T) \) and consequently \( R_i(p^{*}(D)) \geq R_i^{th} \) for all \( i \in M \), which contradicts the existence of at least one user with \( R_i(p^{*}(T)) < R_i^{th} \). Thus from this and from (8) and (14), we conclude that \( \gamma_i(p^{*}(D)) > \gamma_i(p^{*}(T)) \) for each user \( i \in L \), and \( \gamma_i(p^{*}(D)) = \gamma_i(p^{*}(T)) \) for each user \( i \in M \setminus L \), which proves Part II.

V. APPLICATION OF DTPC IN COGNITIVE RADIO NETWORKS OR MULTI-SERVICE NETWORKS

It is important to employ proper distributed power control schemes for different service requirements (e.g. real-time versus non-real time services) or for different network paradigms (e.g. a network with only primary users versus the one with both primary and secondary users). In this section, we demonstrate that DTPC along with TPC and OPC can be properly employed to apply different priorities of transmission and service requirements.

A. Application of DTPC in Cognitive Radio Networks

Cognitive radio is a promising technique for improving the efficiency of using the scarce frequency spectrum. This technique allows unlicensed (secondary) users to opportunistically access the frequency spectrum concurrently with licensed (primary) users, provided that the interference caused by secondary users would not deny any primary user its throughput requirements. In other words, primary users have a higher priority of transmission than secondary users.

In previous sections, we assumed equal priority of transmission for all users. Now we consider a cognitive radio network in which primary and secondary users coexist. Let us denote the sets of primary and secondary users by \( \mathcal{P} \) and \( \mathcal{S} \), respectively. We adjust the effective interference threshold in DTPC for primary and secondary users so that primary users are protected against all secondary users, and at the same time, secondary users can use the available resources opportunistically. In the following theorem, we show that when the minimum-target-SIR requirements for primary users are feasible, if DTPC is utilized by primary users and OPC is utilized by secondary users, then secondary users do not prevent primary users from hitting their target-SIRs.

**Theorem 5:** Suppose that each primary user applies DTPC with its given effective interference threshold, and each secondary user employs OPC. If the set of minimum-target-SIRs for primary users are feasible, then for any number of secondary users:

I. A unique fixed-point exists, to which the set of transmit power levels for all users converge.

II. All primary users are supported at least with their minimum target-SIRs, i.e. \( \gamma_i(p^{*}(D)) \geq \widehat{\gamma}_i \), and for secondary users we have \( \gamma_i(p^{*}(D)) = \Gamma_i(p^{*}(D)) \).

**Proof:** Since the power-updating function corresponding to either DTPC or OPC is two-sided scalable as shown in Theorem 3 and in [2], respectively, we conclude that if there exists a fixed-point, then it is unique, and to which, the set of transmit power levels for all primary and secondary users converge. Thus, to prove Part I, we need only to show the existence of a fixed-point. This can be proved similar to Theorem 2 by noting that the transmit power levels of secondary users operating in the OPC mode appear as the background noise to primary users, and for the DTPC, the existence of the fixed-point does not depend on the background noise. Part II can be easily proved as follows. Since primary users employ DTPC, thus at the fixed-point (whose existence and uniqueness is guaranteed as shown in Part I), each primary user obtains a SIR equal to \( \gamma_i(p^{*}(D)) \), which may be higher or lower than its minimum target-SIR, i.e. \( \Gamma_i(p^{*}(D)) = \widehat{\gamma}_i \). Thus, since each secondary user employs OPC, a given secondary user obtains a SIR equal to \( \Gamma_i(p^{*}(D)) = \widehat{\gamma}_i \).
• Class II. (real-time data services): sensitive to both bit error rates and delays.
• Class III. (non-real-time data services): sensitive to bit error rates and tolerant to delays.

For a data service that is sensitive to bit error rates, its target-SIR is higher than that of a voice service. For users of voice service, TPC is suitable. In contrast, for data services, a higher SIR results in a better throughput. We consider two types of data services, namely real-time (e.g. online video) or non-real-time (e.g. Email). A real-time data service and voice service have transmission priority as compared to a non-real-time data service, implying that if the system is not feasible, the users in class III should be softly removed (i.e. decrease their target-SIRs) in favor of users in classes I and II. Thus, to satisfy transmission priority and SIR requirements, users in classes I to III should apply TPC, DTPC, and OPC, respectively.

VI. GAME-THEORETIC ANALYSIS OF DTPC

In this section, we deal with selfish users who do not cooperatively follow a predefined strategy (in contrast to what we have assumed so far) unless their own utilities are maximized. In what follows, assuming selfish and non-cooperative users, we investigate how and with what pricing scheme, at the Nash equilibrium (NE) of the power control game, the system throughput is improved similar to DTPC.

Game theory is a powerful mathematical tool to model and analyze such selfish and interactive decision making. In a game theoretic view of the power control problem [4], [13], [14], users are players. Each user \(i \in M\) chooses its strategy by setting its own transmit power level from its strategy space \(P_i = [0, \infty)\), resulting in a utility value that represents the throughput of that user, as well as its associated costs, denoted by \(U_i(p_i, p_{-i})\), where \(p_{-i}\) denotes the transmit power vector for all users except user \(i\). The commonly used concept in solving game theoretic problems is the NE at which no user can improve its utility by unilaterally changing its transmit power.

It is well established that in contrast to the case in which no pricing is applied, the pricing scheme could affect the individual user’s decision in such a way that the efficiency of NE from a given goal’s point of view (e.g. from fairness or system’s point of view) is improved. In [4], for a concave throughput function that is increasing with respect to SIR, we have shown that the SIR-based pricing can be utilized to satisfy some specific goals (such as fairness, aggregate throughput optimization, or trading off between these two goals) at NE. In [2], [3] it was shown that the best response function for the utility function \(U_i(p_i, p_{-i}) = \sqrt{\gamma_i} - \alpha_i p_i\) corresponds to the opportunistic power control scheme. Note that the throughput function defined in [2], [3] has no physical meaning.

A pricing-based utility function for a given user \(i\) in many non-cooperative power control games (NPCGs) is

\[
U_i(p_i, p_{-i}) = T_i(p_i, p_{-i}) - c_i(p_i, p_{-i}), \tag{18}
\]

where \(T_i\) is the function representing user \(i\)'s throughput, and \(c_i\) is its pricing function. We use the logarithmic function of SIR as the throughput defined in (6), but our results can be applied to any other throughput function that is an increasing and concave function of SIR. In what follows, we set up a NPCG with a pricing scheme that is linearly proportional to SIR, and analytically show that with a proper choice of pricing (proportionality constant), its outcome is the same as that of our proposed DTPC, implying that at NE, the system throughput is improved similar to DTPC.

We propose a SIR-based pricing scheme in which the utility function for user \(i\) is

\[
U_i(p_i, p_{-i}) = T_i(p_i, p_{-i}) - \alpha_i \gamma_i(p_i, p_{-i}), \tag{19}
\]

where \(\alpha_i \geq 0\) is the price per unit of the actual SIR at the base station for user \(i\). We will show that this pricing scheme enables us to adequately influence the best response function of each user for improving the throughput (similar to DTPC) by a proper choice of pricing introduced in the following theorem.

**Theorem 6:** In NPCG \(G = (\mathcal{M}, (P_i), (U_i))\) in which \(U_i\) is (19), the best response of user \(i \in M\) to a given power vector \(p_{-i}\) is the same as our proposed power-updating function of DTPC in (14) with the given values of \(\tilde{\gamma}_i\) and \(R_i^h\) if the pricing in (20) for each user \(i\) is set to

\[
\alpha_i = \begin{cases} 
\frac{k R_i^2}{R_i^2 + \tilde{\gamma}_i R_i^h}, & \text{if } R_i \leq R_i^h \\
\frac{k}{1 + \tilde{\gamma}_i}, & \text{if } R_i > R_i^h,
\end{cases} \tag{20}
\]

where \(k = \frac{W}{\sqrt{P}}\). Thus, a unique NE for this game exists, and is the fixed point of DTPC, i.e. the fixed-point of DTPC denoted by \(p^*\) is the solution to

\[
\max_{p_i \in P} T_i(p_i, p_{-i}^*) - \alpha_i \gamma_i(p_i, p_{-i}^*), \text{ for all } i \in M. \tag{21}
\]

**Proof:** To obtain the best response function denoted by \(b_i(p_{-i})\), we use the first and the second derivatives of the pricing-based utility with respect to \(p_i\)

\[
\frac{\partial U_i}{\partial p_i} = \frac{k}{R_i} \left( \frac{1}{1 + \tilde{\gamma}_i} - \alpha_i \right), \tag{22}
\]

\[
\frac{\partial^2 U_i}{\partial p_i^2} = -\frac{k}{R_i^2 (1 + \tilde{\gamma}_i)^2}. \tag{23}
\]

For a given \(R_i\), we note from (22) that \(\partial U_i/\partial p_i = 0\) has a unique root, that is \(p_i = \frac{R_i^{th^2}}{R_i^h} \tilde{\gamma}_i\) if \(R_i \leq R_i^{th}\), and is \(p_i = \frac{R_i^{th^2}}{R_i^h} \tilde{\gamma}_i R_i\) if \(R_i > R_i^{th}\). From (23), we note that \(\partial^2 U_i/\partial p_i^2 < 0\), which means the unique root of \(\partial U_i/\partial p_i = 0\) globally maximizes \(U_i\). Thus the best transmit power in response to \(p_{-i}\) that maximizes \(U_i\) is also unique and is given by

\[
b_i(p) = \begin{cases} 
\frac{R_i^{th^2}}{R_i^h} \tilde{\gamma}_i, & \text{for } R_i \leq R_i^{th} \\
\frac{R_i^{th^2}}{R_i^h} \tilde{\gamma}_i R_i, & \text{for } R_i > R_i^{th}
\end{cases} \tag{24}
\]

which is the same as in DTPC, and its fixed point is the NE for \(G = (\mathcal{M}, (P_i), (U_i))\). As there exists a unique fixed point in DTPC, the NE exists and is unique. Thus, the fixed-point of DTPC is the unique solution to (21).

Note that the proposed pricing in (20) implies that when the effective interference for a given user is below the threshold,
its pricing is increased as its effective interference is increased in order to discourage selfish users from transmitting at high power levels until a threshold for the effective interference is reached upon which, its pricing is fixed.

Another important property of the proposed pricing scheme is that it depends on the information pertinent to that user only. This enables each user to compute its pricing in (20) in a distributed and iterative manner by

$$\alpha_i(t+1) = \begin{cases} \frac{kp_i(t)^2}{p_i(t)^2 + \gamma_i(t)\gamma_i R_{th}i^2}, & \text{if } \frac{p_i(t)}{\gamma_i(t)} \leq R_{th}i \\ \frac{1}{1 + \gamma_i}, & \text{if } \frac{p_i(t)}{\gamma_i(t)} > R_{th}i \end{cases},$$

which is obtained from (20) by using $R_i(p(t)) = p_i(t)/\gamma_i(t)$. This is in contrast to the pricing schemes developed in [13], [14] that require the base station to announce the pricing to each user.

VII. SIMULATION RESULTS

Now we provide simulation results to get an insight into how our proposed DTPC improves the system throughput while guaranteeing a minimum value for the individual user’s throughput. We assume a processing gain of 100. The AWGN power at the receiver, i.e., $\sigma^2$, is assumed to be -113 dBm. As in [14], we adopt a simple model $h_i = kd_{s_i}^{-4}$ for the path-gain where $d_{s_i}$ is the distance between user $i$ and its base station $s_i$, $k$ is the attenuation factor that represents power variations, and take $k = 0.09$. In our simulations, for DTPC we take the target-SIP $\eta_i = 1$ (as in [2], [15]) for all users. The values of $\hat{\gamma}_i$ are the same in TPC and in DTPC.

We first consider a single-cell to investigate how our proposed algorithm works as compared to TPC and OPC. Then we consider multi-cell networks, first a two-cell network to investigate how DTPC deals with users’ movements, and finally proceed to 4-cell networks each with a different number of users for two separately simulated cases (with and without a cognitive network). In this way, we see the performance of DTPC in multi-cell wireless networks for different number of users for a primary network only, and for a cognitive (secondary) radio network.

A. Single-Cell Network

Consider 4 users indexed from 1 to 4 with the same minimum target-SIR of 13 dB in a single-cell environment where their distance vector is $d = [300, 530, 740, 860]^T$ m, in which each element is the distance of the corresponding user from the base station. Figs. 2-4 show the transmit-power and the received SIR in each iteration (power-updating step) for users when TPC, OPC, and DTPC are applied, respectively. In TPC (Fig. 2), all users reach their minimum target-SIRs. In OPC (Fig. 3), the user with the best channel (user 1) transmits at high power and obtains a high SIR, but the other users obtain very low SIRs, much lower than their minimum target-SIR. As we see in Fig. 4, in our proposed DTPC, in contrast to OPC, the minimum target-SIR for each user is guaranteed, and the system throughput is notably improved as compared to TPC.
power levels according to DTPC, TPC, and OPC, the same time. As stated in Section V-B, in accordance with minimum acceptable target-SIRs of users are not reachable at target-SIR.

\[ \sum_{i=1}^{4} \gamma_{i} = 0 \]

\[ \gamma_{i} = 10 \log_{10} \left( \frac{P_{i}}{N_{0}} \right) \]

 Specifically, by using DTPC, user 1 which has the best channel utilizes the additional available resource (power) and obtains a SIR of 20 dB which is 7 dB higher than its minimum target-SIR by dynamically increasing its target-SIR, and at the same time other users hit their minimum target-SIRs, i.e. 13 dB.

Now consider that the above 4 users are running different services with different target-SIRs. Suppose that user 3 utilizes a voice service for 6 seconds with \( \gamma_{3} = 13 \text{ dB} \), users 1 and 2 utilize a real-time data service with \( \gamma_{2} = \gamma_{4} = 15 \text{ dB} \) for 2 and 4 seconds, respectively, and user 4 utilizes a non-real-time data service with \( \gamma_{3} = 18 \text{ dB} \) for 6 seconds. All users are active from the start-time \( t = 0 \) and each user stops its transmission immediately after its respective activity time elapses, i.e. users 1-4 are active in time-intervals of \([0, 6], [0, 2], [0, 4], \) and \([0, 6] \), respectively. Note that when all users are active, the system is infeasible (because \( \sum_{i=1}^{4} \frac{\gamma_{i}}{\gamma_{i} + g_{i}} = 1.03 > 1 \)), meaning that the minimum acceptable target-SIRs of users are not reachable at the same time. As stated in Section V-B, in accordance with the users’ service requirements, users 1-4 set their transmit power levels according to DTPC, TPC, and OPC, respectively. Each user updates its transmit power every 1 ms (1 KHz). Fig. 5. shows the transmit-power and the received SIR versus time for users 1-4. Note that for the time interval \([0, 2] \) seconds during which all users are active, the voice-service user obtains its target-SIR, the two real-time data service users, i.e. users 1 and 2 obtain SIRs equal to and higher than their minimum target-SIRs, respectively, and the non-real-time data service user obtains the low SIR of -22.76 dB. During this interval, the three real-time (voice and data) service users are prioritized with respect to the non-real time data service user. When user 1 leaves the system (at \( t = 2 \)), the interference is decreased, the remaining real-time data user (i.e. user 2) and the non-real time data service user (i.e. user 4) immediately increase their transmit power and obtain higher-SIRs while the voice service user continues to receive its target-SIR. Similarly, when user 2 leaves the system (\( t = 4 \)), user 4 obtains an acceptable SIR. Such prioritization is applied in a distributed and automatic manner, as each user employs an appropriate distributed power control algorithm in accordance with its service and priority requirements.

To show that similar improvements in system throughput are obtained for different snapshots of users’ locations and for different values of target-SIRs, we consider a single-cell wireless network with a radius of 250 m and with 10 fixed users. The minimum target-SIRs are considered the same for all users, ranging from 2 dB to 10 dB with a step size of 0.5 dB. For each target-SIR, we average the corresponding values of the system throughput at equilibrium, where the algorithm converges for both TPC and DTPC, for 1500 independent snapshots from a uniform distribution of users’ locations within a single-cell.

Fig. 5. Transmit power and SIRs for each user with a different service in DTPC.

Fig. 6. Average system throughput per Hz per user (\( \sum_{i=1}^{4} \frac{\log_{2}(1 + \gamma_{i})}{W/M} \)) obtained by TPC and DTPC vs. target-SIR.

Fig. 6. Average system throughput per Hz per user, i.e. \( \sum_{i=1}^{4} \frac{\log_{2}(1 + \gamma_{i})}{W/M} \) obtained by TPC and DTPC versus target-SIRs. The average system throughput obtained by applying OPC is 50. Although this is very high as compared to TPC and DTPC, it does not guarantee an average SIR for all users, and only users with better channels obtain acceptable SIRs.

In Fig. 6 we observe that DTPC results in an improved system throughput compared to TPC, as was proved in Theorem 4. At the same time, all users are supported by utilizing DTPC at least with their minimum target-SIRs, which is not the case when OPC is employed. As expected, increasing the minimum acceptable target-SIRs for users results in decreasing the difference between system throughputs of DTPC and TPC. The reason is that when the minimum acceptable target-SIR \( \gamma_{i} \) for each user \( i \) is increased, then \( \sum_{i} \frac{\gamma_{i}}{\gamma_{i} + g_{i}} \) is increased and the feasible system approaches an infeasible system, meaning that the users’ target-SIRs cannot be increased further, because otherwise we may have \( \sum_{i} \frac{\gamma_{i}}{\gamma_{i} + g_{i}} > 1 \), i.e. the system becomes infeasible. In other words, for high values of minimum target-SIRs (e.g. values close to 10 dB in our simulation), after providing all users with their minimum target-SIRs, the remaining resources are not sufficient to further improve the system throughput. For instance, for the target-SIR of 10 dB, we have \( \frac{\gamma_{i}}{\gamma_{i} + g_{i}} = 0.91 \), which means very little is available.
B. Multi-Cell Networks

1) Two-Cell Networks: To show how DTPC works when users move, we assume a two-cell wireless network with 9 users as shown in Fig. 7. The minimum target-SIR for all users is 13 dB. Suppose that users 1 to 6, 8, and 9 are fixed, and user 7 at \( t = 0 \) starts moving from the starting point \( x = 700 \) m in cell No. 2 towards the end-point \( x = 300 \) in cell No. 1 along the illustrated line at a uniform speed of 20 m/s (72 km/h).

As we observe in Fig. 8, for the interval that user 7 is the closest user to the base station in cell No. 2 (i.e., \([0, 2]\)) it obtains the highest SIR (greater than its minimum acceptable target-SIR), and the other users in that cell, including user 8, receive their minimum acceptable target-SIRs. At the same time, in cell No. 1, user 2 obtains the highest SIR and the other users in that cell obtain their minimum acceptable target-SIRs. As user 7 moves farther from base station No. 2 and user 8 becomes the closest user to that base station, user 8 increases its transmit power and obtains a SIR higher than its minimum acceptable target-SIR, and user 7 decreases its transmit power and hits its minimum acceptable target-SIR. When user 7 enters cell No. 1 (i.e. at \( t = 10 \) seconds), as the number of users in that cell is increased (in contrast to cell No. 2 in which the number of users is decreased), the system throughput for cell Nos. 1 and 2 are decreased (as per the reduced SIR for user 2) and increased (as per the increased SIR for user 8), respectively. Finally when user 7 becomes the closest user to base station No. 1, it obtains the highest SIR in cell No. 1, whereupon user 2 receives its minimum acceptable target-SIR. These show that DTPC works well when users move. This is achieved by dynamically assigning high target-SIRs to user(s) with better channels, and assigning at least the minimum acceptable SIR to the remaining users, in a distributed manner.

2) A 4-Cell Primary Network with and without a Cognitive Radio Network: Now we consider a 4-cell wireless network to investigate the performance of DTPC in multi-cell networks and in cognitive radio networks. We first consider a 4-cell network without any secondary user (all users are primary and
Fig. 11. Average transmit-power and average SIRs for 4 secondary users (the sum of transmit power consumed and sum of SIRs received by secondary users divided by the total number of secondary users, respectively) versus the number of primary users, when the primary and the secondary users employ DTPC and OPC, respectively.

have equal priority) and then consider the same 4-cell primary network but along with a secondary (cognitive radio) network. An example of such a 4-cell wireless network and a cognitive radio network is shown in Fig. 9.

The primary users are distributed in an area 1000 m × 1000 m covered by 4 base stations each covering a 500 m × 500 m cell. The minimum acceptable target-SIR for each primary user is 10 dB. We consider the range of 1 primary user/cell (a total of 4 users) to 9 primary users/cell (a total of 36 users) with a step size of 4 users. For each snapshot, TPC and DTPC are separately applied and their corresponding system throughput is computed at the equilibrium. We average the corresponding values of the system throughput for 1500 snapshots of uniform distribution of users’ locations. Fig. 10 shows the average system throughput per Hz per user (the sum of throughput for all primary users divided by the allocated bandwidth and the corresponding total number of users) versus the total number of users, for TPC and DTPC. As can be seen, DTPC outperforms TPC in terms of system throughput.

Now we consider the same primary network together with a single base-station (located at the center of the whole area) with 4 secondary users (who coexist with primary users). In this scenario, primary users employ DTPC and secondary users employ OPC as discussed in Section V. Fig. 11 shows the average transmit power and SIR for each secondary user. Fig. 12 shows the average system throughput for the primary network and for the cognitive radio network versus the total number of primary users. Note that as the number of primary users increases, secondary users sense it by experiencing an increased effective interference at their receiver and consequently reduce their transmit power levels in accordance with OPC, resulting in lower SIRs and a lower system throughput. Note that by comparing the average system throughput achieved for the two cases (with and without the cognitive radio network shown in Figs. 10 and 12, respectively), we see that secondary users do not prevent primary users from obtaining their expected system throughput.

VIII. CONCLUSIONS

We studied the problem of system throughput optimization subject to a given lower bound for the users’ SIRs in wireless cellular networks. To address this problem in a distributed manner, we introduced our proposed DTPC. In the proposed DTPC, when the effective interference for a given user is less than a given threshold, that user dynamically sets its target-SIR which is a decreasing function of the effective interference, to a value higher than the minimum acceptable target-SIR; otherwise it keeps its target-SIR fixed at its minimum acceptable value.

We showed that the proposed algorithm converges to a unique fixed point starting from any initial transmit power in both synchronous and asynchronous power updating cases. We also showed that our proposed algorithm not only guarantees the (feasible) minimum acceptable target-SIRs for all users as is the case by TPC (and in contrast to OPC), but also significantly improves the system throughput as compared to TPC. Furthermore, we discussed the application of DTPC along with OPC in cognitive radio networks and in wireless cellular networks with different services. For the case in which users are assumed to be selfish, a game theoretic analysis of our DTPC algorithm was presented via a non-cooperative power control game with a new pricing function. It was shown that the best response function for each user is the same as our proposed power-updating function of DTPC.

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