Distributed Power Allocation for Cooperative 
Access in Cognitive Radios to Guarantee 
QoS for Cell Edge Primary Users

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Abstract—Cognitive radio (CR) systems have been proposed for efficient usage of spare spectrum licensed to primary systems. This leads to the issue of providing as much spectrum to CR users as possible while not degrading the quality of service (QoS) of primary users of the spectrum. This paper proposes a novel cooperation scheme between primary and CR users to guarantee QoS of primary users up to the cell edge while making the licensed spectrum available for opportunistic access by the CR users. We suggest that the primary users at the cell edge, who have poor QoS, should allow secondary users to access their spectrum, while at the same time, the secondary users would help to enhance the primary users QoS using superposition coding on the primary users transmissions. Thus the proposed method can provide a so called “win-win strategy” by benefiting both primary and CR users. The proposed cooperative access scheme in cognitive radios solves efficiently the sum-rate maximization problem on cognitive Gaussian Multiple Access Channels (GMACs) for power allocation of primary systems that cooperates with CR systems in a distributed fashion. We solved the problem using iterative Jacobian method in a distributed manner. Numerical results show that the QoS of primary users at the cell edge is improved by the proposed cooperative access scheme.

I. INTRODUCTION

Cognitive radio (CR) technology offers potentials to make efficient use of unused spectrum, by enabling secondary users to opportunistically access spectrum that is currently unused [1], while yielding the spectrum to the primary users when access by primary users is detected. Recently, some work [2] - [4] has studied the achievable rate of CR systems from an information theoretic point of view. In the seminal work [3], the achievable rate of a single CR user is provided under the constraints that (i) there is no interference to the primary user, and (ii) the primary users are oblivious to the presence of CR users. In [4], the results of [3] were extended to the case with multiple CR users in the presence of primary transmission. These CR users wish to communicate with a same CR base station (BS). Focusing on the Gaussian noise case, they define this system as cognitive Gaussian multiple access channels (GMACs) and characterize its achievable rate region. Reference [4] suggests a method that allows primary and CR users to communicate simultaneously under mutual interference without causing any quality of service (QoS) degradation to the primary users. In this method, however, incentive mechanisms to encourage primary users to allow secondary CR users to access the spectrum have not been considered.

In wireless cellular networks, guaranteeing the user’s QoS at the cell boundary usually also guarantees the QoS of all other users in the cell. Providing such QoS guarantees is usually a difficult problem to solve. Additionally, finding unused spectrum for the use of CR users is another hard problem in the design of CR systems. This paper considers a method to guarantee QoS for primary users near the cell edge, by means of cooperation of CRs that allow secondary users to access the primary users spectrum. In addition, this paper proposes an incentive mechanism to encourage primary users to make spectrum available to CR users. By extending [4], we suggest that the primary users at the cell edge, who have poor QoS, should allow secondary users to access their spectrum, while at the same time, the secondary users would help to enhance the primary users QoS using superposition coding on the primary users transmissions. By using a simply designed cooperation method, we show that the primary users could enhance their QoS while at the same time making it possible for secondary users to obtain the spectrum for stable transmissions as a result of the superposition coding scheme. Therefore, this proposed cooperative access scheme in CR can provide a so called “win-win strategy” by benefiting both primary and CR users. To the best of our knowledge, this is the first work to simultaneously solve the problems of provisioning QoS the cell edge for...
primary users and provisioning stable spectrum for CR users in the secondary system [9].

For simultaneous transmissions of primary users and secondary CR users, in [4], they solve the relaxation of power allocation problem for nonconvex sum-rate maximization on cognitive GMAC to result in a nonconvex quadratic optimization problem, whose solution is difficult to get. Thereby, they rather than handle the nonconvexity through a heuristic scheme that use Lagrange multiplier. Reference [8] uses a conceptually simple semidefinite relaxation (SDR) to solve the same problem instead of using a heuristic algorithm. The solution in [8] can be achieved more easily in polynomial time by applying SDR. However, the power allocation schemes used in [4] and [8] are not practical because they are centralized method which are not appropriate for a decentralized CR systems. We solve the same problem efficiently using iterative Jacobian method in distributed way [13]. We show that this distributed algorithm for power allocation is converged to a fixed point and numerical results show that QoS of cell edge users is improved by the cooperation of CRs while secondary users receive more opportunities to transmit in the licensed band.

The rest of this paper is structured as follows. Section II presents a model of the CR multiple access channel (MAC). Section III investigates the sum-rate maximization on GMACs. Section VI explains how QoS for cell edge users are guaranteed by cooperation of CRs. Section V introduces distributed power allocation for cooperative access in cognitive radios to guarantee QoS for cell edge primary users. Numerical results are presented in Section VI to illustrate the effectiveness of our proposal. Section VII draws some conclusions.

II. MODELING OF COGNITIVE RADIO MULTIPLE ACCESS

Figure 1 describes the system model [4]. This model has a primary mobile station (MS) communicating with a primary base station (BS) and multiple secondary MSs, who desire access to a secondary BS using primary frequency bands without license. In Figure 1, $g_p$ is the path gain from the primary MS to the primary BS, $g_k$ $(k = 1, 2, ..., N)$ are the path gains of the interference links from the respective secondary MSs to the primary BS, $h_k$ $(k = 1, 2, ..., N)$ are the path gains from the respective secondary MSs to the secondary BS, and $h_p$ is the path gain of the interference link from the primary MS to the secondary BS. The average power of the primary MS is $p_p$ and the average power of the $k$-th secondary MS is assumed to be $p_k$ $(k = 1, 2, ..., N)$.

In order to meet the CRs transmit requirements such as (i) no interference is created for the primary system, and (ii) the primary MS-BS link is oblivious to the presence of secondary MSs employing CRs, a scenario of only one secondary MS was studied in [3] by introducing a superposition coding method. The maximum rate of the CR is achieved by a mixed strategy of dirty paper coding and cooperation with the primary MS. The optimal strategy for superposition coding for secondary MS $k$ is as follows:

$$X_k^n = \hat{X}_k^n + x_k \sqrt{\frac{p_k}{p_p}} X_p^n, \quad k = 1, 2, ..., K.$$  \hspace{1cm} (1)

where the first term $\hat{X}_k^n$ denotes the codeword when the secondary MS $k$’s codeword is dirty-paper coded based on the primary MS’s codeword $m_p$, and the second term, the cooperation part, is the duplicate information of the primary MS, which is combined with the information from the primary MS. In addition, $x_k^2$ is the power ratio of the cooperation part. Figure 2 shows a simple superposition coding scheme for primary MS and secondary MS using QPSK constellation. From the figure, it can be seen that the superposition coding signal of CR MS is the sum of the primary MS signal and CR MS signal.

With the above superposition coding strategy, the cognitive multiple access channel can be regarded as a GMACs with the common interference noncausally available at each transmitter. Based on Theorem 2 in [6], the capacity region of the later is equal to the capacity region of the standard GMACs without the common interference, which gives an achievable rate region of the cognitive multiple access channel:

$$C_{\text{mac}}(h, p, x) =$$

$$\left\{ R : R(S) \leq \frac{1}{2} \log \left( 1 + \frac{\sum_{k=1}^{N} h_k p_k (1 - x_k^2)}{N_x} \right) \right\}$$

$$\forall S \subset \{1, 2, ..., N\}$$  \hspace{1cm} (2)

Fig. 1. Path gain model of CR multiple access

Fig. 2. Superposition coding
where $h_k$ is the path gain from secondary MS $k$ to the secondary BS, $N_s$ is additive white Gaussian noise at the secondary BS, $\mathbf{h} = (h_1, h_2, ..., h_N)^T$ and $\mathbf{p} = (p_1, p_2, ..., p_N)^T$ are constants, and $\mathbf{x} = (x_1, x_2, ..., x_N)^T$ is a variable vector. The data rate of the primary MS in the absence of any secondary MS is,

$$R_p = \frac{1}{2} \log(1 + \frac{g_p p_p}{N_p})$$

where $g_p$ is the path gain from the primary MS to the primary BS, and $N_p$ is the additive white Gaussian noise at the primary BS. At the primary BS’s receiver, its data rate in the presence of secondary MSs using superposition coding can be expressed as follows:

$$R_p' = \frac{1}{2} \log \left( 1 + \frac{\left( \sqrt{g_p p_p} + \sum_{k=1}^{N} \sqrt{g_k p_k x_k} \right)^2}{N_p + \sum_{k=1}^{N} g_k(1 - x_k^2)p_k} \right)$$

where $g_k$ is the path gain from secondary MS $k$ to the primary BS. Eqn. (4) should be greater than the original rate (3) as if there were no interference from the secondary MSs, i.e., the following inequality must be satisfied:

$$\log \left( 1 + \frac{g_p p_p}{N_p} \right) \leq \log \left( 1 + \frac{\left( \sqrt{g_p p_p} + \sum_{k=1}^{N} \sqrt{g_k p_k x_k} \right)^2}{N_p + \sum_{k=1}^{N} g_k(1 - x_k^2)p_k} \right)$$

$$\frac{g_p p_p}{N_p} \leq \frac{\left( \sqrt{g_p p_p} + \sum_{k=1}^{N} \sqrt{g_k p_k x_k} \right)^2}{N_p + \sum_{k=1}^{N} g_k(1 - x_k^2)p_k}$$

We formulate a sum-rate maximization problem under the constraint (5). We are interested in the maximum sum-rate of the CR network is expressed as follows:

$$R_s = \frac{1}{2} \log \left( 1 + \frac{\sum_{k=1}^{N} h_k p_k (1 - x_k^2)}{N_s} \right)$$

III. SUM-RATE MAXIMIZATION ON COGNITIVE GMAC

This section formulates a sum-rate maximization problem within the achievable rate region, whose solution will be the power allocation to each secondary MS that maximizes the sum-rate. The formulation is made as a nonconvex quadratic problem for cognitive GMAC [8], where the mixed strategy of dirty-paper precoding and cooperation with the primary MS is adopted. The solution then determines the allocation of each secondary MSs power ratio for the cooperation part, $x_k^2$ in $p_k$. maximize $\sum_{k=1}^{N} h_k p_k (1 - x_k^2)$

subject to $\frac{g_p p_p}{N_p} \leq \frac{\left( \sqrt{g_p p_p} + \sum_{k=1}^{N} \sqrt{g_k p_k x_k} \right)^2}{N_p + \sum_{k=1}^{N} g_k p_k (1 - x_k^2)}$

$0 \leq x_k \leq 1$ for all $k$.

where $N_p$ is the additive white Gaussian noise at the primary BS, $\mathbf{g} = (g_1, g_2, ..., g_N)^T$, $\mathbf{h} = (h_1, h_2, ..., h_N)^T$ and $\mathbf{p} = (p_1, p_2, ..., p_N)^T$ are constants, and $\mathbf{x} = (x_1, x_2, ..., x_N)^T$ is a variable vector.

Reference [4] does not observe that (7) could be formulated as a standard type of quadratically constrained quadratic problem (QCQP). Instead [4] provides a heuristic and iterative search that requires high complexity. First, this new paper changes (7) into a standard QCQP minimization problem. (See the details in the top of the next page.)

minimize $\mathbf{x}^T \mathbf{H} \mathbf{x} + \mathbf{z}$

subject to $\mathbf{x}^T \mathbf{G} \mathbf{x} + 2 \mathbf{b}^T \mathbf{x} + c \leq 0$

$0 \leq \mathbf{x} \leq 1$,

where, $\mathbf{H} = \text{diag}(h_k p_k)$ is an $N \times N$ diagonal matrix, and $\mathbf{z} = -\sum_{k=1}^{N} h_k p_k$. The problem (8) is a QCQP, but unfortunately $\mathbf{G}$ is not positive semi-definite, which makes the problem nonconvex. In general, a nonconvex QCQP is hard to solve. However, using semi-definite relaxation (SDR), the solution can be obtained with much lower complexity. The problem (8) could be formulated as following.

minimize $\text{Tr}(\mathbf{H} \mathbf{X}) + \mathbf{z}$

subject to $\text{Tr}(\mathbf{G} \mathbf{X}) + 2 \mathbf{b}^T \mathbf{x} + c \leq 0$

$$\begin{bmatrix} \mathbf{X} & \mathbf{x} \\ \mathbf{x}^T & 1 \end{bmatrix} \succeq 0,$$

$0 \leq \mathbf{x} \leq 1$,

which is an SDP problem. This is called the SDP relaxation of the standard nonconvex QCQP. Since it is an SDP problem, it is easy to solve. In addition, we provide a Lagrangian relaxation of sum-rate maximization on cognitive GMAC in Appendix I.

IV. PROVIDING QOS GUARANTEE TO CELL EDGE USERS USING COOPERATIVE CRs

This section proposes a method to guarantee QoS for cell edge users using cooperation of CRs employed by secondary MSs for spectrum access. The QoS of cell edge users may be...
QoS enhancement of cell edge users using CRs as follows.

Therefore, we formulate a new power allocation problem for user’s QoS, we set the target SNR, \( \gamma^{(target)} \), of primary MSs as below,

\[
\frac{(\sqrt{g_p p_p} + \sum_{k=1}^{N} \sqrt{g_k p_k} x_k)^2}{N_p + \sum_{k=1}^{N} g_k p_k (1 - x_k^2)} \geq \gamma^{(target)} \geq \frac{g_p p_p}{N_p}
\]

Therefore, we formulate a new power allocation problem for QoS enhancement of cell edge users using CRs as follows,

\[
\text{maximize } \sum_{k=1}^{N} h_k p_k (1 - x_k^2) \\
\text{subject to } \frac{(\sqrt{g_p p_p} + \sum_{k=1}^{N} \sqrt{g_k p_k} x_k)^2}{N_p + \sum_{k=1}^{N} g_k p_k (1 - x_k^2)} \geq \gamma^{(target)} \\
0 \leq x_k \leq 1 \text{ for all } k.
\]

We changes (26) into a standard QCQP minimization problem. (See the details in the top of the next page.)

\[
\text{minimize } x^T H x + z \\
\text{subject to } x^T \tilde{G} x + 2\tilde{b}^T x + \tilde{c} \leq 0
\]

very poor because of low path gain and interference from adjacent cells. In this paper, we suggest that secondary MSs have the possibility to relay the traffic of the primary system towards the intended destinations. Therefore, a secondary MS can help a primary link to increase the signal-to-noise ratio (SNR) at the receiver using superposition coding, while transmitting its own data to the secondary BS. In order to guarantee cell edge user’s QoS, we set the target SNR, \( \gamma^{(target)} \), of primary MSs as below,

\[
\frac{(\sqrt{g_p p_p} + \sum_{k=1}^{N} \sqrt{g_k p_k} x_k)^2}{N_p + \sum_{k=1}^{N} g_k p_k (1 - x_k^2)} \geq \gamma^{(target)} \geq \frac{g_p p_p}{N_p}
\]

Therefore, we formulate a new power allocation problem for QoS enhancement of cell edge users using CRs as follows,

\[
\text{maximize } \sum_{k=1}^{N} h_k p_k (1 - x_k^2) \\
\text{subject to } \frac{(\sqrt{g_p p_p} + \sum_{k=1}^{N} \sqrt{g_k p_k} x_k)^2}{N_p + \sum_{k=1}^{N} g_k p_k (1 - x_k^2)} \geq \gamma^{(target)} \\
0 \leq x_k \leq 1 \text{ for all } k.
\]

We changes (26) into a standard QCQP minimization problem. (See the details in the top of the next page.)

\[
\text{minimize } x^T H x + z \\
\text{subject to } x^T \tilde{G} x + 2\tilde{b}^T x + \tilde{c} \leq 0
\]
systems. We suggest a distributed power ratio allocation for CR
for cognitive GMAG by using iterative Jacobian method
Therefore, in this section, we provide a distributed algorithm

\[ -g_p p - \gamma^{(\text{target})} (N_p + \sum_{k=1}^{N} g_k p_k) \leq 0 \]

\[ x^T \tilde{G} x + 2 \tilde{b}^T x + \tilde{c} \leq 0 \]

\[ \tilde{G} = \begin{bmatrix} - (\gamma^{(\text{target})} + 1) g_1 p_1 & - \sqrt{g_1 p_1 g_2 p_2} & \cdots & -\sqrt{g_1 p_1 g N_p N} \\ - \sqrt{g_2 p_2 g_1 p_1} & - (\gamma^{(\text{target})} + 1) g_2 p_2 & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ - \sqrt{g N_p N g_1 p_1} & \cdots & \cdots & - (\gamma^{(\text{target})} + 1) g N_p N \end{bmatrix} \]

\[ \tilde{b} = [ - \sqrt{g_p p g_1 p_1} \cdots - \sqrt{g_p p g N_p N} ]^T \]

\[ \tilde{c} = - g_p p + \gamma^{(\text{target})} (N_p + \sum_{k=1}^{N} g_k p_k) \]

Now, for the purpose of solving (15) in a distributed fashion, consider the following iterative power allocation procedure:

\[ x^{(n+1)} = - \lambda H^{-1} \tilde{G} x^{(n)} - \lambda H^{-1} \tilde{b} \quad (17) \]

or equivalently for each MS \(k\)

\[ x_k^{(n+1)} = \lambda \frac{1}{n_k p_k} \left\{ (\gamma^{(\text{target})}) g_k p_k x_k^{(n)} + \sqrt{g_k p_k} \Lambda^{(n)} \right\} + \sqrt{g_k p_k g_k p_k} \quad (18) \]

where \(\Lambda^{(n)} = \sum_{j=1}^{N} g(j) x_j^{(n)}\). Taking into the power ratio constraint, let us define the mapping,

\[ T(x^{(n+1)}) = \left[ \min \left\{ 1, - \lambda H^{-1} \tilde{G} x^{(n)} - \lambda H^{-1} \tilde{b} \right\} \right]^+ \quad (19) \]

where \([z]^+ = \max \{z, 0\}\). We summarized a distributed power allocation scheme for cooperative access in cognitive radios to guarantee QoS for cell edge primary users in the top of the next page.

\[ (H + \lambda \tilde{G}) x + \lambda \tilde{b} = 0 \quad (14) \]

\[ (I + \lambda H^{-1} \tilde{G}) x + \lambda H^{-1} \tilde{b} = 0 \quad (15) \]

\[ x = - \lambda H^{-1} \tilde{G} x - \lambda H^{-1} \tilde{b} \quad (16) \]

V. DISTRIBUTED POWER ALLOCATION FOR COOPERATIVE ACCESS IN COGNITIVE RADIOS TO GUARANTEE QoS FOR CELL EDGE PRIMARY USERS

The solutions of [4] and SDR [8] are centralized methods. They are not practical for the implementation CR systems which are decentralized. Distributed implementation of power allocation is important in CR systems for scalability reasons. Therefore, in this section, we provide a distributed algorithm for cognitive GMAG by using iterative Jacobian method [13]. We suggest a distributed power ratio allocation for CR systems.

Lagrangian of (11) is defined as,

\[ L(x, \lambda) = x^T (H + \lambda \tilde{G}) x + 2 \lambda \tilde{b}^T x + \lambda \tilde{c} + z \quad (12) \]

The solution of \(L(x, \lambda)\) exists in the following condition,

\[ \frac{\partial L}{\partial x} = 2(H + \lambda \tilde{G}) x + 2 \lambda \tilde{b} x = 0. \quad (13) \]

The problem is the same as solving linear system equation, in order to solve this problem in a distributed fashion, we apply Jacobian algorithm [13].

A. Convergence

In this section, we show that the optimal power allocation ratio will be converged. In the general iterative, let \( \alpha_1, \alpha_2 \ldots \) be the eigenvalue of the iteration matrix \(- \lambda H^{-1} \tilde{G}\) and define \(\rho(- \lambda H^{-1} \tilde{G}) = \max_{k} |\alpha_k|\). The constant \(\rho(- \lambda H^{-1} \tilde{G})\) is called the spectral radius of \(- \lambda H^{-1} \tilde{G}\). The following result guarantees that the optimal power allocation ratio will be found.

**Proposition 1**: For any initial \(x^{(0)}\), the mapping \(T\) converges with geometric rate to a power ratio vector for each MS \(k\), if such power allocation is feasible.

**Proof**: Consider the weighted maximum norm \(\|x\|_{\infty} = \|W x\|_{\infty}\) and consistent weighted matrix norm.
A primary MS at the cell edge broadcasts its SNR, $\gamma^{(\text{target})}$, $g_{\text{p}} p_{k}$, and $N_p$.

A primary BS initializes $\lambda^{(0)}$, and $x_k^{(0)}$ and broadcasts $\lambda^{(0)}$, $x_k^{(0)}$, and $\sum_{j=1}^{N} \sqrt{g_{j} p_{j} x_j^{(0)}}$.

**Secondary MS Power Allocation Algorithm:**

Each MS updates power ratio until it converges.

At time $(t)$

$$x_k^{(n+1)} = \left\{ \begin{array}{ll} 1, & \gamma^{(k)}(g_k) x_k^{(n)} / \sum_{j=1}^{N} g_{j} p_{j} x_j^{(n)} \vspace{1mm} \end{array} \right. $$

The primary BS broadcasts $\Lambda^{(n)} = \sum_{j=1}^{N} \sqrt{g_{j} p_{j} x_j^{(n)}}$.

When $x_k^{(n)}$ converges to $x_k^{(t)}$, MS transmit to primary BS with $x_k^{(t)}$.

**Primary BS Algorithm:**

Compute SINR with received power $x_k^{(t)}$ from all secondary MSs,

If $\gamma^{(\text{target})} > \left( \sum_{k=1}^{N} g_{k} p_{k} x_k^{(t)} \right) / (N_p + \sum_{k=1}^{N} g_{k} p_{k} (1 - x_k^{(t)}))$

Update $\lambda^{(t+1)} = \lambda^{(t+1)} + \Delta$

and broadcast $\lambda^{(t+1)}$ to each MS

Else

Stop

End

$$\| -\lambda^{-1} \tilde{G} \|_{\infty}^{W} = \| W \cdot \lambda^{-1} \tilde{G} \cdot W \|_{\infty}$$

where $W$ is a nonsingular matrix. It can be shown that $-\lambda^{-1} \tilde{G}$ is nonnegative and therefore has a positive real eigenvalue equal to spectral radius to $\rho(-\lambda^{-1} \tilde{G})$ By ([16], Th 3.7), a necessary condition for a feasible solution (17) to exist that $\rho(-\lambda^{-1} \tilde{G}) < 1$ Let $e = (e_i)$ be the Perron-Frobenius eigenvector of $-\lambda^{-1} \tilde{G}$, that is $-\lambda^{-1} \tilde{G} e = \rho(-\lambda^{-1} \tilde{G}) e$ and choose $W = \text{diag} \{1/e_i\}$, then

$$T \left( x^{(n)} \right) - x^* \|_W \leq \| -\lambda^{-1} \tilde{G} x^{(n)} - \lambda^{-1} \tilde{G} b - x^* \|_W \leq \| -\lambda^{-1} \tilde{G} x^{(n)} - \lambda^{-1} \tilde{G} b \| \leq \| -\lambda^{-1} \tilde{G} \|_W \cdot \| x^{(n)} - x^* \|_\infty.$$ 

Therefore,

$$\| x^{(n)} - x^* \|_W \leq \rho(-\lambda^{-1} \tilde{G}) \cdot \| x^{(0)} - x^* \|_\infty$$

so if $\rho(-\lambda^{-1} \tilde{G}) < 1$ and the fixed point is within the feasible power range, $T$ is a pseudo-contraction mapping with respect to the weighted maximum norm and converges to a unique fixed point give by $x^* = T(x^*)$ which is solution to $(I + \lambda^{-1} \tilde{G}) x + \lambda^{-1} b = 0$. Q.E.D.

It is also possible to prove convergence of $T$ by verifying that is a standard interference function, which in addition guarantees its asynchronous convergence. It should be also pointed that the algorithm will converge to a fixed point.

**B. A simple numerical example**

This section provides a simple numerical example of the convergence of power allocation ratio. The example assumes
<table>
<thead>
<tr>
<th>Objective value</th>
<th>1062.8</th>
<th>1062.6</th>
<th>1061.6</th>
</tr>
</thead>
</table>

that the path gain vector of the interference link from secondary MS to primary BS is $g = [10 \ 9 \ 8 \ 7 \ 6 \ 5 \ 7 \ 2 \ 1 \ 3]$ and that the path gain vector from secondary MS to secondary BS is $h = [20 \ 18 \ 16 \ 14 \ 12 \ 10 \ 14 \ 4 \ 2 \ 6]$ and $g_p$ is 1. The average power of primary user is $p_p = 1$. The average power of each secondary user is $p_k = 10$, while the additive white Gaussian noise of primary and secondary BS are $N_p = 1$. Fig. 3 shows the fast convergence of the iterative algorithm of each secondary MS’s power allocation. Table I shows that the distributed implementation achieved almost the same performance as P. Cheng’s method [4] and SDR [8].

VI. SIMULATION RESULTS
We consider a scenario where the primary MS employs IS-95, DS-CDMA to share a common frequency with the secondary CR system. We assume the cell structure as shown in Fig. 4, where a primary hexagonal cell covers small CR hotspots. Each MS’s maximum power is assumed to be 10 mW and bandwidth 1.2288 MHz. The COST 231 Hata urban propagation model is used for the channel gains between BS and MSs:

$$\left\{ \begin{array}{ll}
31.5 + 3.5 \log(d), & \text{if } d > 0.035 \text{ km} \\
31.5 + 3.5 \log(0.035), & \text{if } d \leq 0.035 \text{km} 
\end{array} \right. \quad (21)$$

Lognormal shadowing with mean 0 dB and standard deviation 8 dB is assumed. Thermal noise power is -110 dBm. Other parameters are summarized in Table II.

Table III, IV, V, and VI show some results demonstrating enhancements of cell edge users’ QoS using cooperation of CRs. As Fig. 4 and 5 shows, there exists a primary MS at the boundary of the primary cell and a small secondary hotspot with secondary MSs within the primary cell. In Table III, we assume that there is only one secondary MS. Without cooperation of the secondary MS in the CR system, the primary MS’s SNR is 1.7792 originally. When we set the target SNR of primary user to 1.7792, the superpositioned SNR becomes 1.7792 and the power ratio of cooperation part of secondary MS is 0.2562 and data rate becomes 1.7661 bps. However, if we set the target SNR to 3, which is higher than original SNR (1.7792), the superpositioned SNR becomes 3.0019 but the power ratio of cooperation part in secondary MS is increased to 0.5502 to meet the primary MS’s QoS improvement while secondary MS’s data rate is decreased to 1.2941.

Table IV, V, and VI show the numerical results when the numbers of secondary MSs are 2, 5, and 10 respectively. These numerical results illustrate that the QoS of cell edge users is improved by the cooperation of CRs.

VII. CONCLUSION
We have proposed a QoS enhancement method for users at the cell edge using cooperation of CRs. By using a simply designed cooperation method, the primary MSs could enhance their QoS while the secondary MSs could obtain stable transmissions due to the superposition coding scheme. To our best knowledge, this is the first research to solve the cell edge QoS problem and the problem of obtaining stable spectrum for CR system simultaneously. We have presented numerical results, which show that the QoS of cell edge users is improved by exploiting co-operations of CRs.
TABLE III  
**Numerical Results: 1 Secondary MS’s Case**

<table>
<thead>
<tr>
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<th>Noncooperation</th>
<th>Cooperation</th>
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<tbody>
<tr>
<td>Primary SNR</td>
<td>1.7792</td>
<td>3 (Target)</td>
</tr>
<tr>
<td>Superpositioned SNR</td>
<td>1.7792</td>
<td>3.0019</td>
</tr>
<tr>
<td>Power ratio (x)</td>
<td>0.2562</td>
<td>0.5502</td>
</tr>
<tr>
<td>Secondary MS’s sum-data rate (bps)</td>
<td>1.7661</td>
<td>1.2941</td>
</tr>
</tbody>
</table>

TABLE IV  
**Numerical Results: 2 Secondary MS’s Case**

<table>
<thead>
<tr>
<th></th>
<th>Noncooperation</th>
<th>Cooperation</th>
</tr>
</thead>
<tbody>
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<td>Primary SNR</td>
<td>1.7797</td>
<td>3 (Target)</td>
</tr>
<tr>
<td>Superpositioned SNR</td>
<td>1.7797</td>
<td>3.0001</td>
</tr>
<tr>
<td>Secondary MS’s sum-data rate (bps)</td>
<td>5.1403</td>
<td>4.9329</td>
</tr>
</tbody>
</table>

APPENDIX I: **LAGRANGIAN RELAXATION OF SUM-RATE MAXIMIZATION ON COGNITIVE GMAC**

Lagrangian duality provides lower bounds on the optimal value of the problem and simplifies the computation of the lower bound (or optimal in some cases) on the optimal value of the nonconvex QCQP.

Lagrangian of (8) is,

$$L(x, \lambda, \mu) = x^T (H + \lambda G) x + 2(\lambda b + \frac{\mu}{2})^T x + \lambda c - \mu^T 1 + z,$$

and the dual function is,

$$g(\lambda, \mu) = \inf_{x \geq 0} L(x, \lambda, \mu)$$

$$= \begin{cases} y - w^T (H + \lambda G)^{-1} w & H + \lambda G \succeq 0, \\
-\infty & w \not\in R(H + \lambda G), 
\end{cases}$$

where $y = \lambda c - \mu^T 1 + z$ and $w = \lambda b + \frac{\mu}{2}$.

Then, the use of Schur complement allows the dual problem to be expressed as an SDP:

$$\begin{align*}
\text{maximize} & \quad \phi + \lambda c - \mu^T 1 + z \\
\text{subject to} & \quad \lambda \succeq 0, \quad \mu \succeq 0, \\
& \quad H + \lambda G - \frac{(\lambda b + \frac{\mu}{2})^T}{2} \succeq 0, \\
& \quad \phi, \lambda \in R^N \text{ and } \mu \in R^N.
\end{align*}$$

This result is also known as S-procedure in control theory. The main insight is that while the original problem is possibly nonconvex and numerically hard to solve, its dual can be expressed as an SDP and is easy to solve. Regarding the complexity, it is known that SDR has a complexity order of $O(N^3)$ where $N$ is the number of variables in problems [14]. The number of variables for sum-rate maximization problem on Gaussian cognitive MAC problem addressed in this paper is $N$ which denotes the number of users. While as in the previous approach [4], the problem is solved using a heuristic search algorithm which requires considerations of values (0 or not) for all $x_k$ which leads the complexity order to $O(2^N)$, an NP hard problem.

REFERENCES